Completing the Square

The MEnTe Program

Math Enrichment through Technology

Title V East Los Angeles College

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The easiest quadratic equations to solve are of the type

$$x^2 = r^2$$

where r is any constant.

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EXIT

The solution to $x^2 = r^2$ is

$$x = \pm r$$

The answer comes from the fact that we solve the equation by taking the square root of both sides

$$\sqrt{x^2} = \sqrt{r^2}$$

Since r is a constant, we get

$$\sqrt{r^2} = r$$

where both sides are positive, but since x is a variable it could be negative, yet x^2 is positive and $\sqrt{x^2}$ is also positive.

If we write $\sqrt{x^2} = x$ and x is negative, we are saying that a positive number is negative, which it cannot be.

To get around this contradiction we need to insist that

$$\sqrt{x^2} = |x| = \pm x$$

$\sqrt{x^2} = \pm x$ satisfies all possibilities since if x is positive we use

$$\sqrt{x^2} = x$$

and if x is negative we use

$$\sqrt{x^2} = -x$$

Thus, we get the solution for

$$x^2 = r^2$$

$$\sqrt{x^2} = \sqrt{r^2}$$

$$\pm x = r$$

We generally change this equation by multiplying both sides by ± 1, then we simplify and get

$$x = \pm r$$

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BACK EXIT

We generally skip all the intermediate steps for an equation like

$$x^2 = 16$$

and get

$$x = \pm 4$$

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EXIT

For quadratic equations such as

$$9x^2 = 25$$

we solve and get

$$3x = \pm 5$$

$$x = \pm \frac{5}{3}$$

For quadratic equations that are not expressed as an equation between two squares, we can always express them as

$$ax^2 + bx + c = 0$$

If this equation can be factored, then it can generally be solved easily.

If the equation can be put in the form

$$k(x \pm m)^2 = n^2$$

then we can use the square root method described previously to solve it. The solution for this equation is

$$x = \pm \frac{n}{\sqrt{k}} \mp m$$

The sign of m needs to be the opposite of the sign used in ($x \pm m$)

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The question becomes: "Can we change the equation from the form $ax^2 + bx + c = 0$ to the form $(x \pm m)^2 = n^2$?"

Fortunately the answer is yes!

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The procedure for changing $ax^2 + bx + c = 0$ is as follows. First, divide by a, this gives

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Then subtract $\frac{c}{a}$ from both sides. This gives

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

We pause at this point to review the process of squaring a binomial. We will use this procedure to help us complete the square.

Recall that

$$(x+r)^2 = x^2 + 2rx + r^2$$

If we let

$$\frac{b}{a} = 2r$$

we can solve for r to get

$$r = \frac{b}{2a}$$

Substituting $r = \frac{b}{2a}$ in $(x+r)^2 = x^2 + 2rx + r^2$ we get

$$(x + \frac{b}{2a})^2 = x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$$

Using the symmetric property of equations to reverse this equation we get

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = (x + \frac{b}{2a})^2$$

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Now we will return to where we left our original equation.

If we add $\dfrac{b^2}{4a^2}$ to both sides of $x^2 + \dfrac{b}{a}x = -\dfrac{c}{a}$ we get

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

or $$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

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We can now solve this by taking the square root of both sides to get

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is known as the quadratic formula. It is used to solve any quadratic equation in one variable.

We will show how the quadratic equation is used in the example that follows.

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First, we start with an equation

$$3x^2 + 11x = 20$$

Then we change it to

$$3x^2 + 11x - 20 = 0$$

Remember $ax^2 + bx + c = 0$

From this we get

$$a = 3, b = 11, c = -20$$

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BACK EXIT

We then substitute $a = 3, b = 11, c = -20$ into the quadratic formula, simplify and get our values for x.

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Doing so we get

$$x = \frac{-11 \pm \sqrt{11^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{-11 \pm \sqrt{121 + 240}}{6}$$

$$= \frac{-11 \pm \sqrt{361}}{6}$$

$$= \frac{-11 \pm 19}{6}$$

Remember the quadratic equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and the values are
a = 3, b = 11, c = -20

This gives us two values for x,

$$x = \frac{-11+19}{6} = \frac{4}{3}$$

and

$$x = \frac{-11-19}{6} = -5$$

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BACK EXIT

The equation $3x^2 + 11x = 20$ can be factored into $(3x - 4)(x + 5) = 0$ which will give us the same solutions as the quadratic formula.

However, the beauty of using the quadratic formula is that it works for ***ALL*** quadratic equations, even those not factorable (and even when $b^2 - 4ac$ is negative).

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BACK EXIT

To review—the steps in using the quadratic formula are as follows:

1. Set the equation equal to zero, being careful not to make an error in signs.

$$3x^2 + 11x = 20$$

$$3x^2 + 11x - 20 = 0$$

2. Determine the values of a, b, and c after the equation is set to zero.

$$\underline{3}x^2 \underline{+11}x \underline{-20} = 0$$

$$a = 3, b = 11, c = -20$$

Remember $ax^2 + bx + c = 0$

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3. Substitute the values of a, b, and c into the quadratic formula.

$$a = 3, b = 11, c = -20$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(3)(-20)}}{2(3)}$$

4. Simplify the formula after substituting and find the solutions.

$$x = \frac{-11 \pm \sqrt{11^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{-11 \pm \sqrt{121 + 240}}{6}$$

$$= \frac{-11 \pm \sqrt{361}}{6}$$

$$= \frac{-11 \pm 19}{6}$$

$$x = \frac{-11+19}{6} = \frac{4}{3} \qquad x = \frac{-11-19}{6} = -5$$

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